

COMMENT ON "COLLECTIVE EFFECTS IN BREMSSTRAHLUNG IN PLASMAS"

Carlos A. Iglesias

Lawrence Livermore National Laboratory

PO Box 808, Livermore, CA 94550

Abstract - In a recent study Tsytovich *et al.* [*J. Plasma Phys.* **56**, 127(1996)] claimed to obtain new expressions for electron-ion bremsstrahlung in plasmas. It is shown, however, that they interpreted earlier work incorrectly and simply reproduced well-known results.

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Recently, Tsytovich *et al.* (1996, hereafter TBDF) proposed a new expression for electron-ion bremsstrahlung in a plasma incorporating transition bremsstrahlung. However, radiation processes in plasmas have received considerable attention and the TBDF result already exists in the literature (e.g., Dawson & Oberman 1963; Bekefi 1966). Furthermore, it is shown that their result can be readily derived from standard plasma fluctuation theory (e.g., Ichimaru 1973). For simplicity a classical, non-relativistic, fully ionized two-component plasma in thermal equilibrium is considered. Charge neutrality guarantees that the electron and ion number densities are related by $n_e = Zn_i$ where Z is the ion charge.

The main result in TBDF for electron-ion inverse bremsstrahlung for photons with energy $h\omega$ involves the integral given in their Eq. (4.10)

$$g_{coll}(\omega) = \frac{\sqrt{3}}{\pi} \int_0^\infty \frac{dx}{x} e^{-x^2} \left| 1 + 2 \frac{\omega_{pe}^2}{\omega^2} x^2 W(x) \right|^{-2} \left[\frac{\omega^2 + 2\omega_{pe}^2 x^2}{\omega^2 + 2(1+Z)\omega_{pe}^2 x^2} \right] \quad (1)$$

where W is the plasma function (see Eq. 4.5 of TBDF) and $\omega_{pe} = \sqrt{4\pi e^2 n_e / m_e}$ is the electron plasma frequency with m_e the electron mass. In Eq. (1) the lower limit was adjusted for absorption and the overall constant chosen to agree with the usual gaunt factor (Bekefi 1966).

Contrary to the claims by TBDF, Eq. (1) is not new but rather a well-known result obtained by Dawson & Oberman (1963, see Eq. 18). It is also given by Bekefi (1966, see Eq. 5.25) in a review of plasma radiation processes. In Eq. (1) the term

$$\left| 1 + 2 \frac{\omega_{pe}^2}{\omega^2} x^2 W(x) \right|^{-2} = \left| 1 + \frac{k_e^2}{q^2} W\left(\frac{\omega}{q} \sqrt{\frac{m_e}{2kT}}\right) \right|^{-2} = |\varepsilon(q, \omega)|^{-2}, \quad (2)$$

with $k_e = \sqrt{4\pi e^2 n_e / kT}$ the inverse electron Debye length and kT the plasma temperature in energy units, represents dynamic screening of electrons scattering from ions. The absence of ion contributions in Eq. (2) is due to the photon frequencies of interest, $\omega \gg \omega_{pi} = \omega_{pe} \sqrt{m_e / m_i}$ where m_i the ion mass, so that the ions are assumed to be at rest. The factor

$$\left[\frac{\omega^2 + 2\omega_{pe}^2 x^2}{\omega^2 + 2(I+Z)\omega_{pe}^2 x^2} \right] = \frac{q^2 + k_e^2}{q^2 + (I+Z)k_e^2} = S_i(q) \quad (3)$$

accounts for correlations between the stationary ions. The right hand sides of Eqs. (2) and (3) use $x^2 = m_e \omega^2 / 2kTq^2$ (see Eq. 4.9 of TBDF) where q is the momentum transfer and serve to define $\epsilon(q, \omega)$ and $S_i(q)$ to be used below.

Unfortunately, TBDF compared their results with Eq. (9.132) of Ichimaru (1973) which neglects ion correlations (see Eq. 9.131b). However, a more general result is given in Eq. (9.130) of Ichimaru (1973) that leads to the following gaunt factor,

$$g_{Ich}(\omega) = \frac{1}{8Z^2 e^4 n_i n_e} \sqrt{\frac{6kT}{\pi m_e}} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \int \frac{d\vec{q}}{(2\pi)^3} q^2 V_{ei}^2(q) \times \{S_{ee}(-q, \omega - \omega') S_{ii}(q, \omega') - S_{ei}(-q, \omega - \omega') S_{ei}(q, \omega')\} \quad (4)$$

where V_{ei} is the Fourier transform of the electron-ion Coulomb interaction and $S_{\sigma'\sigma}(q, \omega)$ are the dynamic structure factors defined by the density-density time-correlation functions.

Further progress can be made by using the weakly-coupled approximations that give for photon energies $\omega \gg \omega_{pi}$,

$$S_{ee}(q, \omega) \approx \frac{2n_e q^2 \text{Im} \chi_e(q, \omega)}{\omega k_e^2 |\epsilon(q, \omega)|^2} + \frac{n_i S_i(q) |\chi_e(q, \omega)|^2}{|\epsilon(q, \omega)|^2} \delta(\omega), \quad (5)$$

$$S_{ii}(q, \omega) \approx n_i S_i(q) \delta(\omega), \quad (6)$$

$$S_{ei}(q, \omega) = S_{ie}^*(q, \omega) \approx \frac{\sqrt{Z} n_i S_i(q) [\chi_e^*(q, \omega) + |\chi_e(q, \omega)|^2]}{|\epsilon(q, \omega)|^2} \delta(\omega), \quad (7)$$

where Eqs. (2) and (3) define the dielectric function and ion structure factor, and

$$\chi_e(q, \omega) = \frac{k_e^2}{q^2} W\left(\frac{\omega}{q} \sqrt{\frac{m_e}{2kT}}\right). \quad (8)$$

With Eqs. (5) through (8), Eq. (4) simplifies to

$$\begin{aligned}
g_{Ich}(\omega) &= \frac{I}{8\pi Z^2 e^4 k_e^2 \omega} \sqrt{\frac{6kT}{\pi m_e}} \int \frac{d\vec{q}}{(2\pi)^3} \text{Im} \chi_e(q, \omega) \left| \frac{q^2 V_{ei}(q)}{\epsilon(q, \omega)} \right|^2 S_i(q) \\
&= \frac{\sqrt{3}}{\pi} \int_0^\infty \frac{dq}{q} \exp\left\{-\frac{m_e \omega^2}{2kT q^2}\right\} \left| 1 + \frac{k_e^2}{q^2} W\left(\frac{\omega}{q} \sqrt{\frac{m_e}{2kT}}\right) \right|^{-2} \left[\frac{q^2 + k_e^2}{q^2 + (1+Z)k_e^2} \right] \quad (9) \\
&= g_{coll}(\omega)
\end{aligned}$$

where the last equality follows with the change of variable $x^2 = m_e \omega^2 / 2kT q^2$ and demonstrates that the electron-ion bremsstrahlung expression in TBDF can be readily obtained from plasma fluctuation theory in standard textbooks.

Finally, solar opacity models typically do not use the result in Eq. (9) but make further approximations. On the other hand, the opacity models correct for electron degeneracy neglected in the discussion above. Collective effects in inverse bremsstrahlung were recently examined and found to produce small changes to the solar interior opacity (Iglesias & Rose 1996) where it was also shown that the dominant corrections are included in current opacity models.

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